LAGRANGIAN-EULERIAN MULTISCALE DATA ASSIMILATION IN PHYSICAL DOMAIN WITH TWO-LAYER QUASI GEOSTROPHIC MODEL

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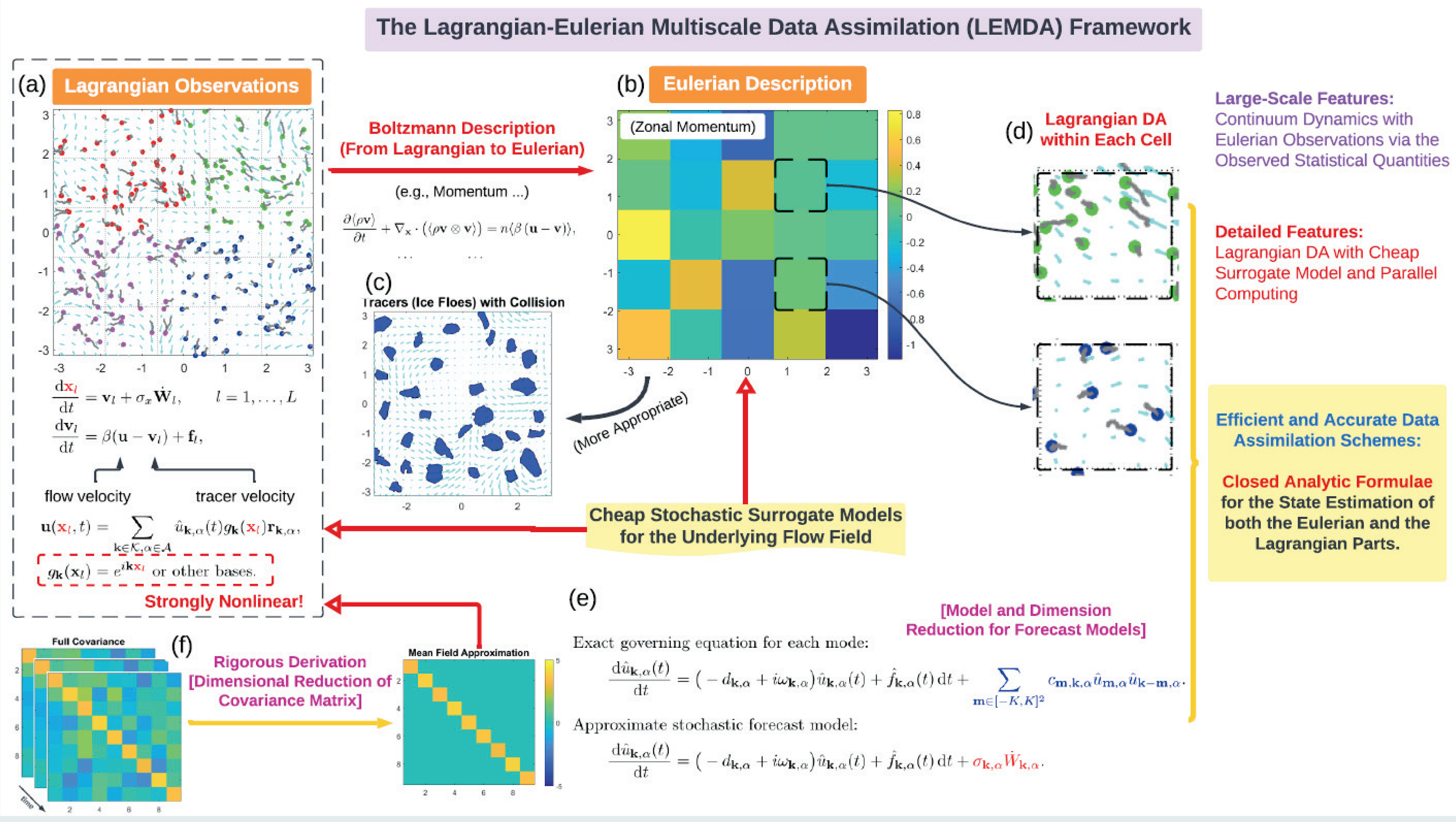
BACKGROUND

What is data assimilation?

It combines observational data with pre-existing physical or computer models to improve predictions of complex systems, like space and weather forcasts or ocean currents. By updating model forecasts with real-time data, it enhances accuracy and reduces uncertainty in simulations.

What is LEMDA?

It is a data assimilation framework that utilises both Lagrangian and Eulerian approach. This means it keeps a track of a set of indivudal particles' projection (Lagrangian) as well as over each grid cell (Eulerian)



ABSTRACT

This research aims to further investigate the process of LEMDA by replacing the Fourier space with the physical domain. Such change in the perspective of domain introduces the advantages of being able to deal in non-periodic system and more intuitive representation of localised phenomena or time-dependent problems. The context of the domain for this paper was set as sea ice floe trajectories to recover the streamline function (Ψ) and potential vorticity (q) in the Arctic regions, which led the model to be derived from two-layer Quasi geostrophic (QG) model. The numerical solution to this model utilises the Conditional Gaussian Nonlinear System (CGNS) to accommodate the inherent non-linearity and continuity in analytical manner. The root mean square error (RMSE) and pattern correlation (Corr) are used to evaluate the performance of the recovered posterior mean of the model. The result corroborates the effectiveness of exploiting the two-layer QG model in physical domain.

MAIN EQUATIONS

CGNS Framework

$$d\mathbf{X} = [\mathbf{A}_0(\mathbf{X},t) + \mathbf{A}_1(\mathbf{X},t)\mathbf{Y}]\,dt + \mathbf{B}_1(\mathbf{X},t)d\mathbf{W}_1(t)$$
$$d\mathbf{Y} = [\mathbf{a}_0(\mathbf{X},t) + \mathbf{a}_1(\mathbf{X},t)\mathbf{Y}]\,dt + \mathbf{b}_2(\mathbf{X},t)d\mathbf{W}_2(t)$$

Two-layer QG Model

$$\frac{\partial q}{\partial t} + \frac{\partial \psi}{\partial x}\frac{\partial q}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial q}{\partial x} = 0 \qquad (3)$$

$$q_i = \nabla^2 \psi_i + \beta y + \frac{k_d^2}{2}(\psi_j - \psi_i) \qquad (4)$$

Benchmarking (compared against analytic solution)

Algorithm 1 Two-layers QG numerical solver
▷ All computations take place for both layers

Require: {All the parameters}
  init constants, time variables and mesh constraints
  init compute the invariant matrix A before time marching
  init ψ^1 and q^1 given the initial ψ
  for $t = 1 : N_t - 1$ **do**
    compute q^{t+1} by solving Equation 3
    compute ψ^{t+1} by solving Equation 4
  end for
return $\psi^{1:N_t}$ and $q^{1:N_t}$

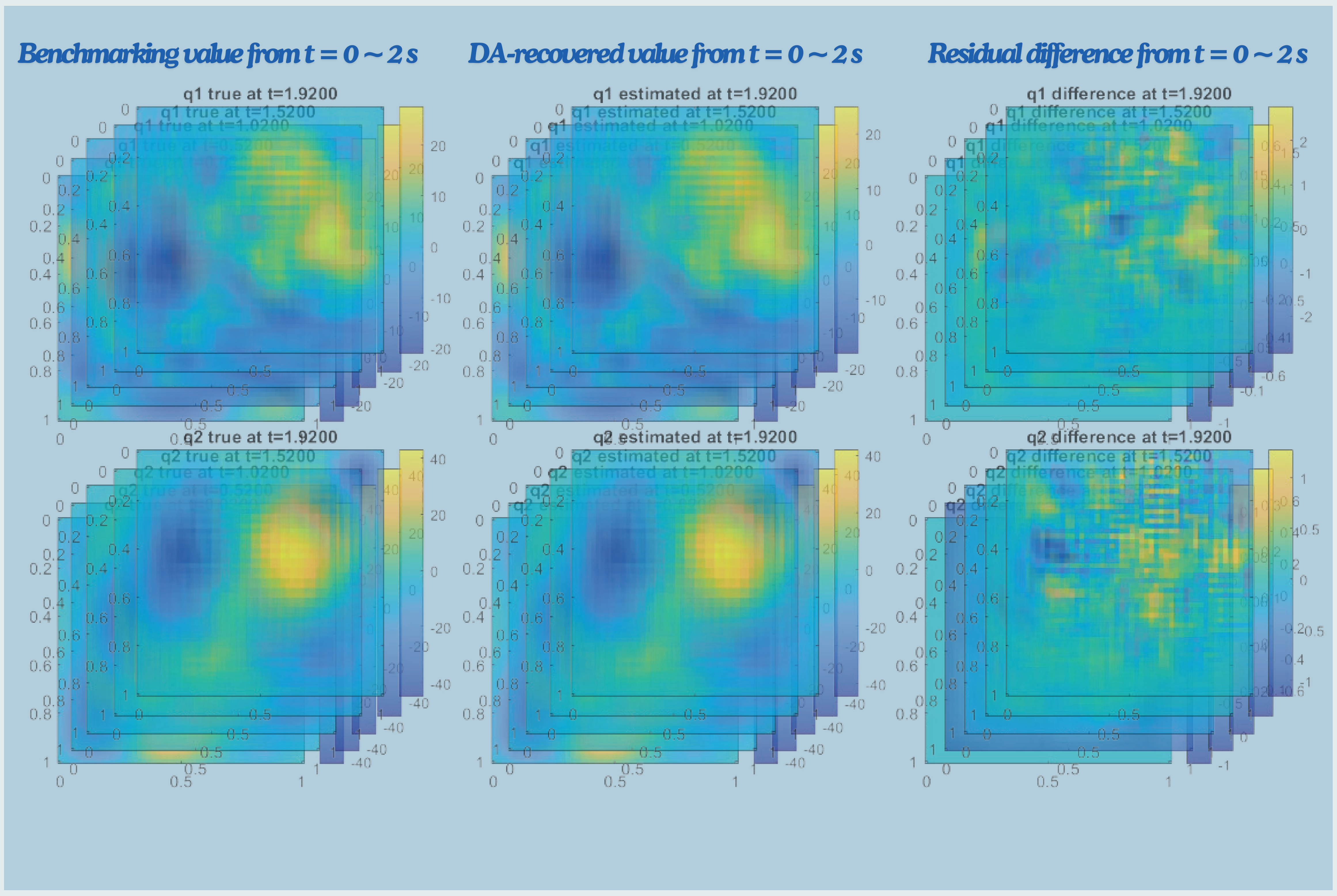
Check out the simulation videos!

The Data Assimilation Model

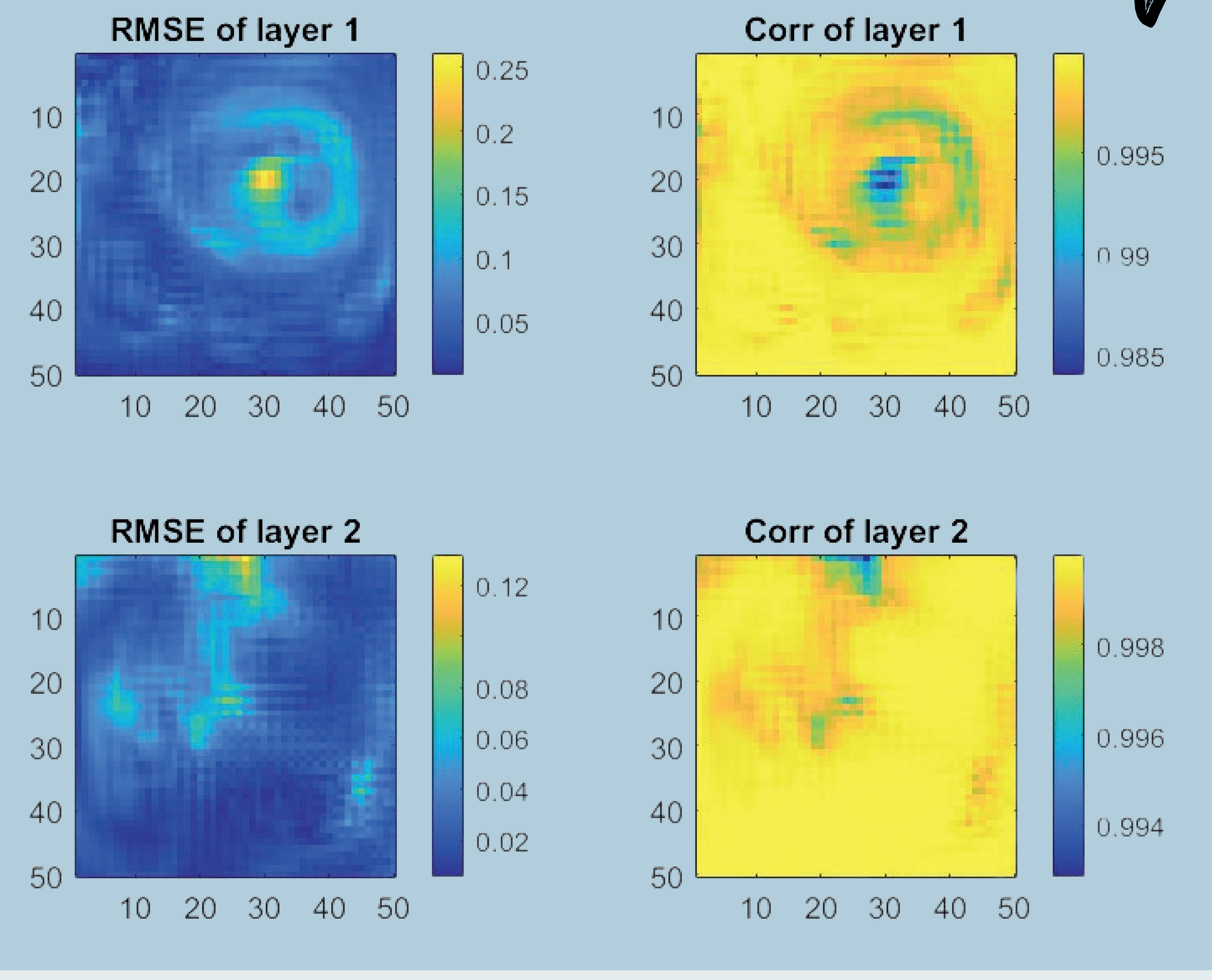
Algorithm 2 Two-layers QG CGNS solver
▷ All computations take place for both layers

Require: {All the parameters}
  init constants, time variables and mesh constraints
  init compute the invariant matrix A before time marching
  init ψ^1, q^1 and μ_f for both layers given the initial ψ
  for $t = 1 : \frac{N_t}{100} - 1$ **do**
    compute q^{t+1} by solving Equation 3
    compute ψ^{t+1} by solving Equation 4
    update $\Sigma\psi$ and Σq for R_f
  end for
  init R_f based on $\psi^{1:\frac{N_t}{100}}$, $\Sigma\psi$, $q^{1:\frac{N_t}{100}}$ and Σq
  for $t = \frac{N_t}{100} : N_t$ **do**
    compute observational noise
    compute ψ^{t+1} via equation 2 with μ_f^t
    find A_0, A_1, a_0 and a_1 by using both Equations 3 and 4
    compute μ_f^{t+1} via CGNS framework
  end for
return $\psi^{1:N_t}$ and $\mu_f^{1:N_t}$

Check out the code artefacts!



ANALYSIS



The performance of DA via CGNS was great overall, as the RMSE ranged from 0 to 0.4 and pattern correlation ranged within 0.99x.

SO, WHAT'S NEXT?

- Neural Network on each cell to achieve accelerated parallel computation
- Recovery of Ψ and q from solely observing the displacement of particles
- Find of analytical convergence condition of Two-layers QG CGNS Solver
- Use of Two-layer QG model as an inductive step for more layers